Instabilities and nonlinear evolution of waves in geophysical fluids and plasmas

Synopsis

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Introduction:

Plasma is widely recognized as the fundamental state of matter comprising a fully or partially ionized quasineutral gas that exhibits collective behavior [1]. Exploring different kinds of nonlinear phenomena in various plasma environments [2] is one of the most interesting and challenging areas in modern physics, as well as applied mathematics. Currently, many researchers emphasize the investigation of wave dynamics in geophysical fluids and plasma media, which connects the microscopic interactions of ionized gases with the macroscopic behavior of planetary systems.

Due to a planet's rotation, gravity, and density stratification, a diverse taxonomy of waves is generated in geophysical systems. Those are gravity waves (buoyancy as a restoring force), acoustic-gravity waves (AGWs) (interaction of pressure and buoyancy forces) [3], and Rossby waves (variation of the Coriolis effect) etc. While in plasma, this complexity is reflected through different modeling approaches depending on the scale of observation. Plasma is treated as a single-fluid medium in magnetohydrodynamics (MHD) to study the wave dynamics of Alfvén waves, drift waves, and magnetosonic waves. The interaction of nonlinearity, dispersion, rotation, stratification, and electromagnetic effects formulates the analytical frameworks and evolution of these waves.

The stability of these nonlinear waves is often fragile. As small perturbations can give rise to different kinds of Instabilities, like modulational instability, parametric instability, shear-driven instability, and wave-wave interactions, which lead to significant wave deformation and an excessive redistribution of energy throughout the medium [4]. As these instabilities develop, nonlinear effects become dominant, resulting in the formation of coherent structures like solitons, wave steepening, shock-like structures, turbulence, and energy cascades across scales [5]. Throughout our research, these are the key findings to understand large-scale circulation, weather variability, ocean mixing, and climate dynamics in geophysical references [6], as well as heating, transport, and stability in laboratory and space plasmas [7].

In 1953, Subrahmanyan Chandrasekhar derived the first analytical solution for a rotating layer of fluid influenced by temperature [8]. A complete theory of Instabilities associated with the Rayleigh-Bénard dynamics causing convection is presented in Chandrasekhar's book. It was shown that the existence of the Cori-

olis force has an inhibitive effect on convection. Zhong and Roberts developed the asymptomatic solutions for a rotating layer of fluid heated from below [9]. Petuviashvilli also obtained an analytical solution for an inviscid layer of fluid subject to the Coriolis force in 1980 [10]. His solution is the first soliton solution that accounts for the sustenance of the vortex structure of a rotating layer of fluid. Primarily, we studied the impact of vortical motions on convective instability in Earth's troposphere. Heating of fluid from below and vorticity of fluid have been taken into account in deriving the system of governing equations [11]. One of the most important applications of this study is to model large-scale and long-lived vortex structures in planetary atmospheres, such as the mysterious problem of the long-lived vortex structure of the Great Red Spot of Jupiter, which has been observed for more than 300 years. Furthermore, we are also interested in investigating the wave propagation and nonlinear phenomena in plasma environments.

Methodology:

To study the convective instability of gravity waves in stratified atmospheric fluids and the nonlinear evolution of acoustic-gravity and internal waves, we use the fluid model consisting of the continuity equation, the momentum balance equation, and the pressure law. Additionally, these fluid models are to be coupled with Maxwell's equations for conducting fluids such as plasmas in the Ionosphere and solar atmosphere. Typically, for the linear stability analysis, we follow the Fourier formalism to derive the linear dispersion relation and the growth rate of fluid density fluctuations. We also consider the effects of temperature-dependent density inhomogeneity and the Coriolis force to study the wave evolution and instabilities. The analytical results are to be validated with numerical analysis and simulation of the nonlinear evolution equation for gravity-driven solitary vortices.

Objectives:

The main objectives of this study are
1. The significance of this research to continue systematic studies on the nonlinear evolution and its application in geophysical reference. Also, we developed the analytic frameworks and numerical simulations for this model.
2. Understanding how nonlinear waves react in geophysical fluids and plasma environments can help to create different weather forecasting models. Those models are used to enhance prediction systems of weather and climate dynamics.
3. Collective interactions of plasma particles with electromagnetic (EM) fields are highly nonlinear. These nonlinear interactions of finite amplitude intense electro-

magnetic waves (EMWs) and relativistic or non-relativistic plasmas have become an important research topic in recent years.

Future research plans:

Our future research will focus on exploring the complex nonlinear evolution of atmospheric waves and the resulting vortex structures. We will examine the stability analysis of gravity waves in stratified fluids and plasmas. Additionally, we will explore a new paradigm of MHD waves and instabilities to offer a more unified theory of wave-wave and wave-particle interactions in geophysical fluids and plasmas.

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